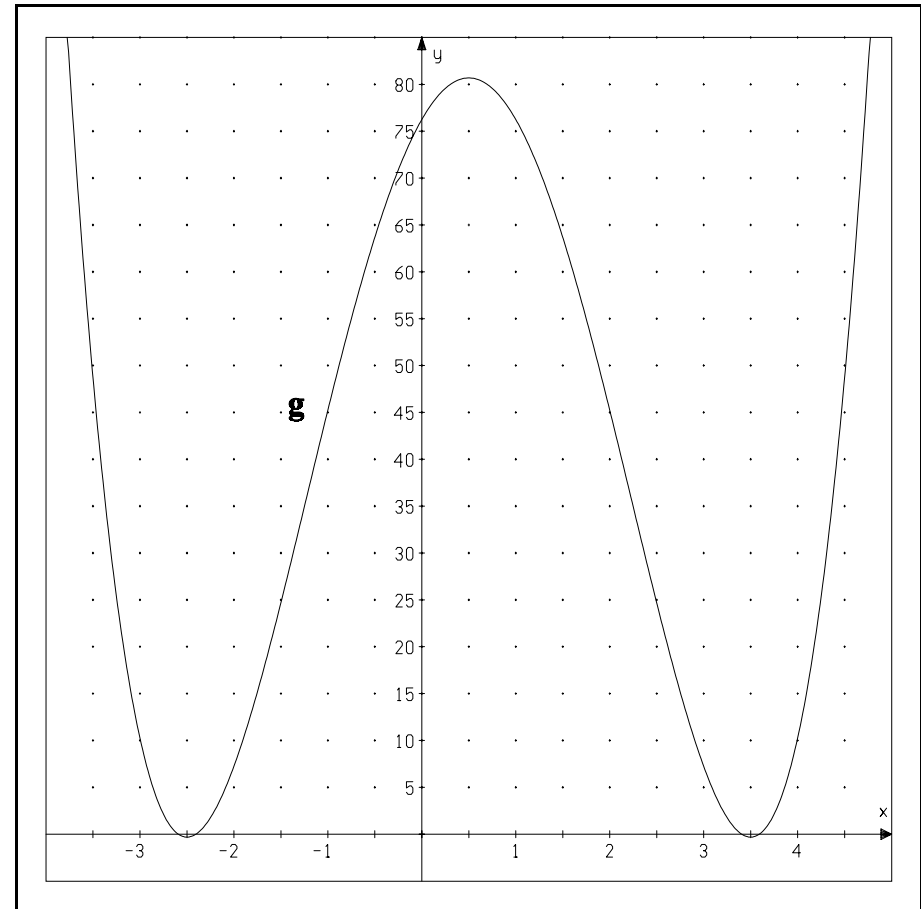
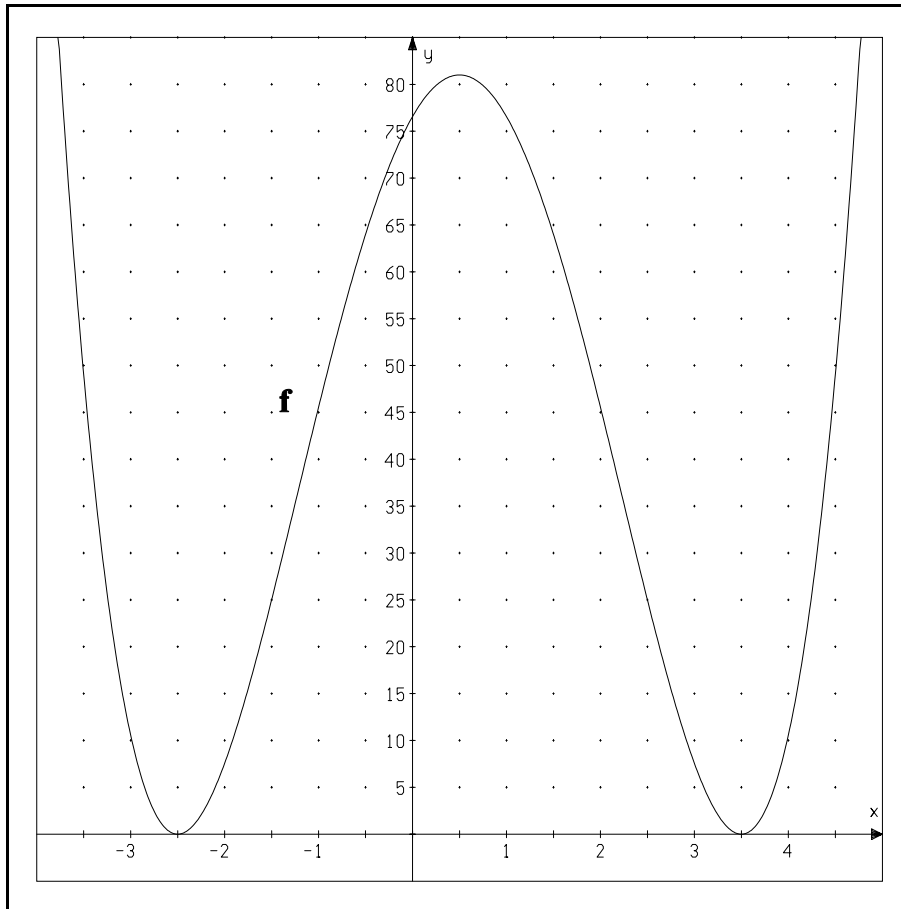


Zur verallgemeinerten Regula Falsi



Dargestellt sind die Funktionen **f** und **g** mit $f(x) = x^4 - 2 \cdot x^3 - \frac{33}{2} \cdot x^2 + \frac{35}{2} \cdot x + \frac{1225}{16}$ und $g(x) = f(x) - \frac{5}{16}$ über $[-4;5]$.

Die Suche nach einem Teilintervall $[a;b]$ mit $f(a) \cdot f(b) < 0$ bleibt hier erfolglos, obwohl Nullstellen vorliegen (hier in Form von Berührungspunkten). Auch wenn Nullstellen sehr nah beieinander liegen, wird man in der Praxis bei der Suche nach einem Teilintervall $[a;b]$ mit $g(a) \cdot g(b) < 0$ auf Schwierigkeiten stoßen.

Man gibt deshalb die Bedingung: ‘unterschiedliche Vorzeichen der Funktionswerte’ auf, und versucht iterativ mit der Rekursionsformel:

$$x_{n+2} = x_{n+1} - \frac{f(x_{n+1}) \cdot (x_{n+1} - x_n)}{f(x_{n+1}) - f(x_n)}, \quad (n \in \mathbb{N}) \text{ zum Ziele zu kommen. (Probleme bei verschiedenen Wahlen von Anfangswerten } x_1 \text{ und } x_2 \text{ ?!)}$$

Verallgemeinerte Regula Falsi mit: $f(x) = x^4 - 2 \cdot x^3 - \frac{33}{2} \cdot x^2 + \frac{35}{2} \cdot x + \frac{1225}{16}$.

| n | x_n | x_{n+1} | $f(x_n)$ | $f(x_{n+1})$ | x_{n+2} |
|----|----------|-----------|-----------|--------------|-----------|
| 1 | 1,000000 | 2,000000 | 76,562500 | 45,562500 | 3,469758 |
| 2 | 2,000000 | 3,469758 | 45,562500 | 0,032594 | 3,470810 |
| 3 | 3,469758 | 3,470810 | 0,032594 | 0,030376 | 3,485221 |
| 4 | 3,470810 | 3,485221 | 0,030376 | 0,007824 | 3,490221 |
| 5 | 3,485221 | 3,490221 | 0,007824 | 0,003431 | 3,494127 |
| 6 | 3,490221 | 3,494127 | 0,003431 | 0,001239 | 3,496335 |
| 7 | 3,494127 | 3,496335 | 0,001239 | 0,000483 | 3,497745 |
| 8 | 3,496335 | 3,497745 | 0,000483 | 0,000183 | 3,498605 |
| 9 | 3,497745 | 3,498605 | 0,000183 | 0,000070 | 3,499138 |
| 10 | 3,498605 | 3,499138 | 0,000070 | 0,000027 | 3,499467 |

| n | x_n | x_{n+1} | $f(x_n)$ | $f(x_{n+1})$ | x_{n+2} |
|----|-----------|-----------|------------|--------------|-----------|
| 1 | -5,000000 | -4,000000 | 451,562500 | 126,562500 | -3,610577 |
| 2 | -4,000000 | -3,610577 | 126,562500 | 62,360124 | -3,232328 |
| 3 | -3,610577 | -3,232328 | 62,360124 | 24,307595 | -2,990706 |
| 4 | -3,232328 | -2,990706 | 24,307595 | 10,144423 | -2,817644 |
| 5 | -2,990706 | -2,817644 | 10,144423 | 4,027091 | -2,703715 |
| 6 | -2,817644 | -2,703715 | 4,027091 | 1,597172 | -2,628831 |
| 7 | -2,703715 | -2,628831 | 1,597172 | 0,623441 | -2,580885 |
| 8 | -2,628831 | -2,580885 | 0,623441 | 0,241921 | -2,550483 |
| 9 | -2,580885 | -2,550483 | 0,241921 | 0,093298 | -2,531398 |
| 10 | -2,550483 | -2,531398 | 0,093298 | 0,035863 | -2,519481 |

| n | x_n | x_{n+1} | $f(x_n)$ | $f(x_{n+1})$ | x_{n+2} |
|----|-----------|-----------|-----------|--------------|-----------|
| 1 | 0,000000 | -1,000000 | 76,562500 | 45,562500 | -2,469758 |
| 2 | -1,000000 | -2,469758 | 45,562500 | 0,032594 | -2,470810 |
| 3 | -2,469758 | -2,470810 | 0,032594 | 0,030376 | -2,485221 |
| 4 | -2,470810 | -2,485221 | 0,030376 | 0,007824 | -2,490221 |
| 5 | -2,485221 | -2,490221 | 0,007824 | 0,003431 | -2,494127 |
| 6 | -2,490221 | -2,494127 | 0,003431 | 0,001239 | -2,496335 |
| 7 | -2,494127 | -2,496335 | 0,001239 | 0,000483 | -2,497745 |
| 8 | -2,496335 | -2,497745 | 0,000483 | 0,000183 | -2,498605 |
| 9 | -2,497745 | -2,498605 | 0,000183 | 0,000070 | -2,499138 |
| 10 | -2,498605 | -2,499138 | 0,000070 | 0,000027 | -2,499467 |

Verallgemeinerte Regula Falsi mit: $f(x) = x^4 - 2 \cdot x^3 - \frac{33}{2} \cdot x^2 + \frac{35}{2} \cdot x + \frac{1225}{16}$.

| n | x_n | x_{n+1} | $f(x_n)$ | $f(x_{n+1})$ | x_{n+2} |
|----|----------|-----------|------------|--------------|-----------|
| 1 | 4,000000 | 5,000000 | 10,562500 | 126,562500 | 3,908944 |
| 2 | 5,000000 | 3,908944 | 126,562500 | 6,869111 | 3,846329 |
| 3 | 3,908944 | 3,846329 | 6,869111 | 4,830846 | 3,697927 |
| 4 | 3,846329 | 3,697927 | 4,830846 | 1,504885 | 3,630780 |
| 5 | 3,697927 | 3,630780 | 1,504885 | 0,642858 | 3,580705 |
| 6 | 3,630780 | 3,580705 | 0,642858 | 0,240830 | 3,550708 |
| 7 | 3,580705 | 3,550708 | 0,240830 | 0,094140 | 3,531458 |
| 8 | 3,550708 | 3,531458 | 0,094140 | 0,036000 | 3,519538 |
| 9 | 3,531458 | 3,519538 | 0,036000 | 0,013832 | 3,512100 |
| 10 | 3,519538 | 3,512100 | 0,013832 | 0,005292 | 3,507491 |

| n | x_n | x_{n+1} | $f(x_n)$ | $f(x_{n+1})$ | x_{n+2} |
|----|-----------|-----------|------------|--------------|-----------|
| 1 | -5,000000 | 1,000000 | 451,562500 | 76,562500 | 2,225000 |
| 2 | 1,000000 | 2,225000 | 76,562500 | 36,293094 | 3,329040 |
| 3 | 2,225000 | 3,329040 | 36,293094 | 0,993076 | 3,360099 |
| 4 | 3,329040 | 3,360099 | 0,993076 | 0,672123 | 3,425142 |
| 5 | 3,360099 | 3,425142 | 0,672123 | 0,196730 | 3,452059 |
| 6 | 3,425142 | 3,452059 | 0,196730 | 0,081424 | 3,471066 |
| 7 | 3,452059 | 3,471066 | 0,081424 | 0,029848 | 3,482066 |
| 8 | 3,471066 | 3,482066 | 0,029848 | 0,011509 | 3,488970 |
| 9 | 3,482066 | 3,488970 | 0,011509 | 0,004364 | 3,493186 |
| 10 | 3,488970 | 3,493186 | 0,004364 | 0,001668 | 3,495794 |

| n | x_n | x_{n+1} | $f(x_n)$ | $f(x_{n+1})$ | x_{n+2} |
|----|-----------|-----------|------------|--------------|-----------|
| 1 | 1,000000 | 6,000000 | 76,562500 | 451,562500 | -0,020833 |
| 2 | 6,000000 | -0,020833 | 451,562500 | 76,190773 | -1,242907 |
| 3 | -0,020833 | -1,242907 | 76,190773 | 35,548720 | -2,311829 |
| 4 | -1,242907 | -2,311829 | 35,548720 | 1,196006 | -2,349043 |
| 5 | -2,311829 | -2,349043 | 1,196006 | 0,779603 | -2,418719 |
| 6 | -2,349043 | -2,418719 | 0,779603 | 0,231440 | -2,448136 |
| 7 | -2,418719 | -2,448136 | 0,231440 | 0,095168 | -2,468680 |
| 8 | -2,448136 | -2,468680 | 0,095168 | 0,034945 | -2,480602 |
| 9 | -2,468680 | -2,480602 | 0,034945 | 0,013459 | -2,488069 |
| 10 | -2,480602 | -2,488069 | 0,013459 | 0,005104 | -2,492631 |